Electron Acceleration by Whistler Waves at Collisionless Shocks

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Nonlinear Wave Steepening and Energy Dissipation

Simple Explanation: larger amplitudes propagate faster than smaller amplitudes
Collisional Shock Waves

If steepening is balanced by energy dissipation results in a stable discontinuity called a shock

\[ KE_{up} - KE_{down} \neq 0 \]
Collisionless Shock Waves

→ accelerate particles and are ubiquitous

Cometary Bow shock

Coronal Mass Ejections

Supernova

Red Spider Nebula

LL Ori in the Great Nebula, constellation Orien
Collisionless Shocks: Energy Dissipation Theory

• **Weak Shocks** (e.g., low Mach number)
  • Quasi-Perpendicular Shocks (i.e., $\theta_{Bn} \geq 45^\circ$)
    • dispersive radiation [e.g., *Tidman and Northrop, 1968; Krasnoselskikh et al., 2002*]
    • instability-driven radiation [e.g., *Sagdeev, 1966; Coroniti, 1970*]
  • Quasi-Parallel Shocks (i.e., $\theta_{Bn} < 45^\circ$)
    • dispersive radiation
    • particle reflection [e.g., *Edmiston and Kennel, 1984; Kennel et al., 1985*]

• **Strong Shocks** (e.g., high Mach number)
  • Quasi-Perpendicular Shocks (i.e., $\theta_{Bn} \geq 45^\circ$)
    • dispersive radiation (limited by phase and group velocity of wave)
      [e.g., *Krasnoselskikh et al., 2002*]
    • instability-driven radiation [e.g., *Sagdeev, 1966; Coroniti, 1970*]
    • particle reflection [e.g., *Edmiston and Kennel, 1984; Kennel et al., 1985*]
  • Quasi-Parallel Shocks (i.e., $\theta_{Bn} < 45^\circ$)
    • particle reflection dominates
Driver/Piston (e.g., ICME) compresses plasma and magnetic field.

- Increasing currents increases $|B_0|$
- Changing currents radiate like an antenna

**Dispersive Radiation**

$t = t_i$

$t = t_f$

Dispersive $\rightarrow \omega = \omega(k)$
Collisionless Shocks: Expected vs. Observed

- **Expectations → Dissipation Mechanisms Control Structure**
  - Weak Shocks (e.g., low Mach number)
    - Shock Structure: laminar/step-like-function (i.e., if present, $\delta B/B \sim$ small)
    - Characteristic: stationary
  - Strong Shocks (e.g., high Mach number)
    - Shock Structure: turbulent
    - Characteristic: non-stationary

- **Observations**
  - Shock Structure: turbulent
    - [e.g., Wilson, 2016; Wilson et al., 2007; 2014a,b; 2017]
    - prior "laminar" observations
      - often due to low resolution data
    - nonlinear waves (i.e., $\delta B/B > 10\%$) are ubiquitous
      - regardless of Mach number or plasma beta or shock geometry
      - both low(large) and high(small) frequency(scale)
    - Characteristic: some weak shocks may be non-stationary
Collisionless Shocks: Expected vs. Observed

**Expectations**
- Weak Shocks (e.g., low Mach number)
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    - regardless of Mach number or plasma beta or shock geometry
    - both low(large) and high(small) frequency(scale)
  - Characteristic: not clear
Of the 430 interplanetary (IP) shocks observed by Wind in the CfA database between Jan. 1, 1995 and Mar. 15, 2016, there were 145 satisfying all the following:

- $M_f \geq 1$
- $1 \leq M_A \leq 3$
- $\theta_{Bn} \geq 45$
- $1 \leq N_{dn}/N_{up} \leq 3$
- $\beta_{up} \leq 1$

Of these 145:

- 113 (~78%) show clear whistler precursors
- 17 (~12%) show no evidence of whistler precursors
- 15 (~10%) have ambiguous features that may be or may not be whistler precursors
- 116 (~80%) were undersampled and/or under resolved

$\sigma \left( n, B_o, V_{sw} \right) = \frac{B_o^2}{\mu_o m_i n V_{sw}^2}$

Of low-Mach number, low beta quasi-perpendicular shocks are NOT laminar!
Weak Shocks: Expected vs. Observed

Expected: Laminar
- quasi-perpendicular ($\theta_{Bn} > 45^\circ$)
- low Mach number ($M \leq 3.0$)
- low beta ($\beta \leq 1.0$)

Observed: Turbulent
- $\delta B/B_0 > 50\%$

Dispersive Radiation → Whistler Precursors

Properties (typical of solar wind)
- aka magnetosonic-whistler mode
- right-hand polarized (to $B_o$)
- $\delta B$ in phase with $\delta n$
- $f_{ci} \leq f_{\text{rest}} \lesssim f_{lh}$ (not a hard cutoff)
  - $f_{\text{rest}} \sim 0.1–11$ Hz
- $f_{sc} \sim 0.1–3.0$ Hz (can be higher)
  - $<f_{sc}> \sim 1$ Hz
- $kc/\omega_{pe} \sim 0.02–6$ or $k\rho_{ce} \sim 0.01–3$
  - $\lambda \sim$ few to $>2000$ km
  - from electron-to-ion scales
- $V_{ph} \sim$ few to $>600$ km/s
  - "Taylor hypothesis" not valid

[e.g., Wilson, 2016; Wilson et al., 2017]
Landau/Cerenkov Interactions

Phase Fronts

Black dots are particles trying to stay with the phase fronts
Cyclotron Resonance

Position

Velocity
Magnetosonic-Whistler Waves

- accelerate electrons from thermal (~10's of eV) to suprathermal (>100's of eV) // to Bo
  - for range of typical IPM plasma properties, $V_{ph,//}$ can reach $c$
- strong ion heating _l_ to $B_o$ from thermal (~10's of eV) to suprathermal (>1000 eV)
  - for range of typical IPM plasma properties, $V_{ph,l}$ can reach $c$
Electron Acceleration in Ion Foreshock

- Only 10/30 disturbances showed clear energetic electron enhancements
- Single, isotropic power-law from 10s of eV to 100s of keV
- Highest energy >700 keV (for reference $T_e \sim 10$ eV) [e.g., Liu et al., 2017]

New Results: Highlights

- Previous assumptions: weak shocks should be smooth (i.e., laminar)
  - Not only are shocks not smooth, waves have average max amplitudes:
    - ~40% of background field (largest at ~160%)
    - ~220% of shock ramp amplitude (largest at ~1530%)
- Large amplitudes completely alter assumptions about dynamics and evolution:
  - nonlinear interactions typically start at $\delta B/B \sim 10\%$
  - average max wave amplitudes are $\delta B/B \sim 40\%!!!$
  - a non-laminar transition completely alters:
    - particle dynamics
      - particle heating
      - particle acceleration
    - evolution and propagation (e.g., CME arrival times)
- Waves propagate obliquely to:
  - background magnetic field \(\rightarrow\) more conducive for particle acceleration
  - shock normal vector \(\rightarrow\) can "run away" from shock
    - shock acts like an antenna radiating the waves
  - coplanarity plane \(\rightarrow\) shocks are intrinsically 3D
    - cannot assume 1D or 2D in simulations!!!
- Wave properties are:
  - wavelengths on electron scales \(\rightarrow\) need full PIC simulations to model!
  - phase speeds comparable to solar wind \(\rightarrow\) cannot use Taylor hypothesis!


Extras
(no particular order)
References

New Results: What and Where

1. could resolve several problems
   A. space weather predictions
   B. solar energetic particles
      i. spacecraft and astronauts
   C. astrophysical shocks (remote sensing)
      i. highest energy particles in the universe

Electrons Energized by factors of ~1000-10000

Old Model: particles pre-energized here

New Model: particles pre-energized here
Full nonlinear estimate of energy gain [called relativistic turning acceleration]

Max($\Delta KE$) $\sim 73$ MeV

Energy gain due to acceleration across the wave potential: $\Delta KE \sim |E| \cdot \Delta x$
[assume $(k r_{ce}) \sim 0.2-1.0$]

Max($\Delta KE$) $\leq 4$ keV

$\gamma \left( \omega - \vec{k} \cdot \vec{V} \right) = n \Omega_{ce}$

\{n = 0, 1, 2, ...\}
Poynting Flux
\[ |\delta S| > 300 \mu W m^{-2} \] \([-= 0.3 \text{ ergs s}^{-1} \text{ cm}^{-2}\)]

1) To accelerate PS e\(^-\) to > 1 MeV
2) Need to deposit \( \varepsilon \sim 5 \times 10^{-12} \text{ J m}^{-3} \)
3) Assume: FA-column of particles with \( L \sim 3 \text{ R}_E \)
4) Integrate: \( d\chi = \varepsilon \, dL \) \((\sim 10^{-4} \text{ J m}^{-2})\)
5) \( \Delta t \sim \chi/\langle \delta S \rangle \)

<table>
<thead>
<tr>
<th>( \delta S ) [W m(^{-2})]</th>
<th>( \Delta t ) (1 %)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5 \times 10^{-8} )</td>
<td>~days</td>
<td>Santolik et al., [2010]</td>
</tr>
<tr>
<td>( 3 \times 10^{-4} )</td>
<td>~33.3 s</td>
<td>Wilson III et al., [2011]</td>
</tr>
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</table>

Wilson et al., [2011]
Whistler Mode Waves: Description

Whistler = generic term used to describe a broad range of waves with an even broader range of properties

Dispersive

\[ n^2 = \frac{\omega^2}{\omega(\Omega_{ce} \cos \theta - \omega)} \]

\[ \omega_{pe} = \sqrt{\frac{n_e e^2}{m_e \varepsilon_0}} \]

\[ \Omega_{ce} = \frac{eB_o}{m_e} \]

Broad Range of Frequencies

\[ \Omega_{ci} \leq \omega \leq \Omega_{ce} \]

Oblique Propagation

\[ \theta_{kB} = \cos^{-1} \left( \frac{\hat{k} \cdot B_o}{|B_o|} \right) \]

Can interact with both electrons and ions through:

1. Landau resonance [e.g., Cairns and McMillan, 2005];
2. cyclotron resonance [e.g., Kennel and Petschek, 1966];
3. nonlinear trapping [e.g., Kellogg et al., 2010];
4. etc.
Coordinate Systems

\[ \hat{b} \equiv \frac{\vec{B}_o}{|\vec{B}_o|} \]

Field-Aligned Coordinate (FAC) System

For more details, see: [e.g., Wilson et al., 2013a]

\[ \theta_{kj} = \cos^{-1}\left( \hat{k} \cdot \vec{J} \right) \left/ \left| \vec{J} \right| \right. \]

\( n \) = shock normal vector

\( \vec{B}_o \) = ambient magnetic field vector

Coordinate Basis Vectors

\( b \) = para

\( (b \times n) \) = perp-2

\( (b \times n) \times b \) = perp-1
Magnetosonic-Whistler Dispersion Relations

Krauss-Varban and Omidi, [1991]

\[ \omega^2 \approx k^2 V_A^2 \left( 1 + \cos^2 \theta_{kB} \left( 1 + \frac{k^2 c^2}{\omega_{pi}^2} \right) \right) \]

\[ \omega_{sh} = \omega_{rest} + \vec{k} \cdot \vec{V}_{\text{trans}} \]

\[ \{ \vec{V}_{\text{trans}} = \vec{V}_{\text{sw}} - V_{sh} \hat{n} \} \]
Quantifying Wave Amplitudes

- Filter then detrend to remove offsets
- Determine the convex hull of the vector components
- Calculate the peak-to-peak amplitude of the convex hull
IP Shock Parameters [Upstream Avgs]
250 Satisfying:  \( M_f \geq 1; M_A \geq 1; R_{21} \geq 1; \) and \( \theta_{Bn} \geq 45 \) deg

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Median</th>
<th>Standard Deviation</th>
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<tbody>
<tr>
<td>( N_i [cm^{-3}] )</td>
<td>0.6</td>
<td>35.5</td>
<td>8.6</td>
<td>7.0</td>
<td>5.8</td>
</tr>
<tr>
<td>( B_o [nT] )</td>
<td>1.0</td>
<td>19.0</td>
<td>5.9</td>
<td>5.5</td>
<td>2.9</td>
</tr>
<tr>
<td>( \beta_{up} )</td>
<td>0.02</td>
<td>3.86</td>
<td>0.54</td>
<td>0.40</td>
<td>0.53</td>
</tr>
<tr>
<td>( \Delta B_o [nT] )</td>
<td>0.4</td>
<td>28.5</td>
<td>6.0</td>
<td>4.6</td>
<td>4.5</td>
</tr>
<tr>
<td>( U_{shn} [km/s] )</td>
<td>37</td>
<td>550</td>
<td>142</td>
<td>109</td>
<td>97</td>
</tr>
<tr>
<td>( V_{shn} [km/s] )</td>
<td>9</td>
<td>1164</td>
<td>490</td>
<td>461</td>
<td>169</td>
</tr>
<tr>
<td>( M_A )</td>
<td>1.15</td>
<td>15.61</td>
<td>2.95</td>
<td>2.47</td>
<td>1.79</td>
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<tr>
<td>( M_f )</td>
<td>1.02</td>
<td>6.39</td>
<td>2.20</td>
<td>1.92</td>
<td>1.05</td>
</tr>
<tr>
<td>( \theta_{Bn} [deg] )</td>
<td>45</td>
<td>90</td>
<td>68</td>
<td>68</td>
<td>13</td>
</tr>
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\[ M_j = j^{th} \text{ Mach number} = U_{shn}/V_j \]  \( [f = \text{ fast, } A = \text{ Alfvén}] \)

\[ N_i/N_f = \text{ shock compression ratio} = R_{21} \]

\[ \theta_{Bn} = \text{ shock normal angle} \]

\[ V_{shn} = \text{ shock normal speed [SC frame]} \]

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Assumed \( T_e = T_i [\text{no electron data used}] \)

1 nT = 10 \( \mu \)G

### IP Shock Parameters [Upstream Avgs]

145 Satisfying: \( M_f \geq 1; 1 \leq M_A \leq 3; \beta_{up} \leq 1; 1 \leq R_{21} \leq 3; \) and \( \theta_{Bn} \geq 45 \) deg

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<td>( B_o ) [nT]</td>
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<td>( \beta_{up} )</td>
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<td>( \Delta B_o ) [nT]</td>
<td>0.4</td>
<td>21.4</td>
<td>4.8</td>
<td>3.8</td>
<td>3.3</td>
</tr>
<tr>
<td>( U_{shn} ) [km/s]</td>
<td>39</td>
<td>275</td>
<td>108</td>
<td>98</td>
<td>50</td>
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<tr>
<td>( V_{shn} ) [km/s]</td>
<td>9</td>
<td>976</td>
<td>452</td>
<td>433</td>
<td>124</td>
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<td>( M_A )</td>
<td>1.15</td>
<td>2.98</td>
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<td>2.01</td>
<td>0.49</td>
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<td>( M_f )</td>
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<td>1.61</td>
<td>0.36</td>
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<td>( \theta_{Bn} ) [deg]</td>
<td>46</td>
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**Assumed** \( T_e = T_i \) [no electron data used]

\( 1 \) nT = 10 \( \mu \)G

IP Shock Parameters [Upstream Avgs] (with precursors)

113 Satisfying:  \( M_f \geq 1; 1 \leq M_A \leq 3; \beta_{up} \leq 1; 1 \leq R_{21} \leq 3; \) and \( \theta_{Bn} \geq 45 \) deg

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<td>438</td>
<td>123</td>
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<td>2.00</td>
<td>2.01</td>
<td>0.51</td>
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IP Shock Parameters [Upstream Avgs] (with precursors)
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<th>$\delta B_{pk-pk}$ [nT] Statistics</th>
<th>$X_{min}$</th>
<th>$X_{max}$</th>
<th>$X_{mean}$</th>
<th>$X_{median}$</th>
<th>$X_{stddev}$</th>
</tr>
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<tr>
<td>$Y_{min}$</td>
<td>0.01</td>
<td>0.4</td>
<td>0.08</td>
<td>0.05</td>
<td>0.07</td>
</tr>
<tr>
<td>$Y_{max}$</td>
<td>0.2</td>
<td>13.0</td>
<td>3.0</td>
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<td>$Y_{stddev}$</td>
<td>0.03</td>
<td>2.5</td>
<td>0.6</td>
<td>0.4</td>
<td>0.5</td>
</tr>
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The maximum of the 113 average precursor amplitudes (i.e., each of the 113 precursors has an array of amplitudes)

$X_j = 113$ values of $j^{th}$ one-variable statistics
$Y_j = j^{th}$ statistic of $113 X_j$ values
IP Shock Parameters [Upstream Avgs] (with precursors)
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<th>( \delta B_{pk-pk}/\langle B_o \rangle_{up} ) Statistics</th>
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<tr>
<td>( Y_{\text{min}} )</td>
<td>0.003</td>
<td>0.04</td>
<td>0.01</td>
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<td>0.008</td>
</tr>
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<td>( Y_{\text{max}} )</td>
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</table>

The maximum of the 113 average precursor amplitudes (i.e., each of the 113 precursors has an array of amplitudes)

\( X_j = 113 \) values of \( j^{\text{th}} \) one-variable statistics
\( Y_j = j^{\text{th}} \) statistic of 113 \( X_j \) values
IP Shock Parameters [Upstream Avgs] \((\text{with precursors})\)

113 Satisfying:  \(M_f \geq 1; 1 \leq M_A \leq 3; \beta_{\text{up}} \leq 1; 1 \leq R_{21} \leq 3;\) and \(\theta_{Bn} \geq 45\) deg

<table>
<thead>
<tr>
<th>(\delta B_{\text{pk-pk}}/\Delta B_o) Statistics</th>
<th>(X_{\text{min}})</th>
<th>(X_{\text{max}})</th>
<th>(X_{\text{mean}})</th>
<th>(X_{\text{median}})</th>
<th>(X_{\text{stddev}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y_{\text{min}})</td>
<td>0.004</td>
<td>0.2</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>(Y_{\text{max}})</td>
<td>0.04</td>
<td>15.3</td>
<td>0.8</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>(Y_{\text{mean}})</td>
<td>0.01</td>
<td>2.2</td>
<td>0.1</td>
<td>0.09</td>
<td>0.2</td>
</tr>
<tr>
<td>(Y_{\text{median}})</td>
<td>0.01</td>
<td>1.1</td>
<td>0.08</td>
<td>0.06</td>
<td>0.1</td>
</tr>
<tr>
<td>(Y_{\text{stddev}})</td>
<td>0.006</td>
<td>2.7</td>
<td>0.2</td>
<td>0.08</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The maximum of the 113 average precursor amplitudes (i.e., each of the 113 precursors has an array of amplitudes)

\[
X_j = 113 \text{ values of } j^{\text{th}} \text{ one-variable statistics}
\]

\[
Y_j = j^{\text{th}} \text{ statistic of 113 } X_j \text{ values}
\]
Whistler Mode Waves: Wavenumber and Frequency Estimates

\[ n^2 = \left( \frac{k c}{\omega_{pe}} \right)^2 = \frac{\omega_{pe}^2}{\omega \left( \Omega_{ce} \cos \theta_{kB} - \omega \right)} \]

\[ \omega_{sc} = \omega_{rest} + \vec{k} \cdot \vec{V}_{bulk} \]

\[ 0 = \tilde{V} \kappa^3 + \left( \cos \theta_{kB} - \tilde{\omega}_{sc} \right) \kappa^2 + \tilde{V} \kappa - \tilde{\omega}_{sc} \]

1. Apply bandpass filter defined by \( \Delta f_{sc} \)
2. Use MVA to determine \( k/|k| \), then:
   A. determine \( \theta_{kB} \)
   B. determine \( \theta_{kV} \)
3. Use range of \( f_{sc} \) to solve for range of \( \kappa \)
4. Use \( \kappa \) range to find range of \( \omega_{rest} \)

\[ \Omega_{cs} = \frac{q_s B_o}{m_s} \]
\[ \omega_{ps} = \sqrt{n_s e^2} \]
\{where: \( s = \text{species} \)\}

\[ \kappa = \frac{k c}{\omega_{pe}}, \quad \tilde{\omega}_j = \omega_j \Omega_{ce}, \]

\[ \tilde{V} = V_{bulk} \cos \theta_{kV}, \quad \tilde{V} = \frac{V_{Ae}}{B_o} \]

See Wilson et al., [2013a] for more details
Whistler Mode Waves: Model Frequency Peak and Estimate Resonance Energy

\[
\begin{align*}
E_{\parallel,\text{res}} &= \left( \frac{B_o^2}{\mu_0 n_e} \right) \left( \frac{\Omega_{ce}}{\omega \cos \theta_{kB}} \right) \left( \cos \theta_{kB} - \frac{\omega}{\Omega_{ce}} \right) \left[ m + \frac{\omega}{\Omega_{ce}} \right]^2 \\
&= \begin{cases} 
-1 & \text{for Normal Cyclotron Resonance} \\
0 & \text{for Landau/Cerenkov Resonance} \\
+1 & \text{for Anomalous Cyclotron Resonance}
\end{cases}
\end{align*}
\]

1. Use range of \( \kappa \) and \( \omega_{\text{rest}} \) values to:
   A. get range of \( E_{\parallel,\text{res}} \) values

Note: \( \Delta f \) used is larger than the FWHM for these events
Impact: my \( \Delta E \) estimates are over exaggerated