Abstract / Objectives

Abstract Propagation of energetic particles through magnetized turbulent media is reconsidered using the exact solution of Fokker-Planck equation [1]. It shows that the cosmic ray (CR) transport in weakly scattering media is nondiffusive. Poor understanding of the CR Planck equation [1]. It shows that the cosmic-ray transport obeys its sources and acceleration mechanisms. We present a simplified approximated version of [2] of the exact solution of Fokker-Planck equation that accurately describes a ballistic, diffusive and transdiffusive (intermediate between the first two) propagation regimes. The transdiffusive phase lasts for a (surprisingly) long time, \( t \lesssim 5t_c \) (five collision times), while starting as early as at \( t \sim 0.5t_c \). Since the scattering rate is energy dependent, \( t_c = t_c(E) \), a large part of the energy spectrum propagates neither diffusively nor ballistically. Its treatment should rely on the exact solution. Significant parts of the spectra affected by the heliospheric modulus, for example, falls into this category.

We present a new approximation of an exact Fokker-Planck propagator. It conveniently unifies the ballistic and Gaussian propagators, currently used (separately) in many Solar modulation and other CR transport models. The maximum deviation of the new propagator from the exact solution (at \( t \approx t_c \)) is less than a few percent. The work on the further improvement is ongoing.

Questions to Answer At times much shorter than the collision time, \( t < t_c \), most particles propagate with their initial velocities or their projections on the magnetic field direction, if present. This regime is called the ballistic, or rectilinear propagation. The question then is what happens next, namely at \( t \sim t_c \) but before the onset of diffusion at \( t \gg t_c \). What exactly is the value of the \( t_c \)? When it is safe to switch to the simple diffusive description?

Exact solution of Fokker-Planck equation

\[
\frac{\partial}{\partial t} \psi + \frac{\partial}{\partial x} \left( \psi \frac{d \psi}{d x} \right) = \frac{1}{\sqrt{2 \pi}} \frac{1}{\sigma} e^{-\frac{(x - \mu)^2}{2 \sigma^2}} \frac{\partial}{\partial x} \left( \psi \frac{d^2 \psi}{d x^2} \right)
\]

The Fokker-Planck equation

\[
\frac{\partial f}{\partial t} + \frac{\partial}{\partial x_1} \left[ v f \right] + \frac{\partial^2}{\partial x_1^2} f = \frac{D}{\sigma^2} \left( f - \frac{1}{2} f^2 \right)
\]

Setting \( M = 1 \), the moment-generating function

\[
f(x,t) = \int f(x,t') \frac{e^{\Delta t}}{\sqrt{2 \pi \Delta t}} \, dx = \int f(x,t') \frac{e^{\Delta t}}{\sqrt{2 \pi \Delta t}} \, dx + \frac{1}{\sqrt{2 \pi \Delta t}} \int f(x,t') \frac{e^{\Delta t}}{\sqrt{2 \pi \Delta t}} \, dx
\]

For large \( \Delta t \), the solution becomes diffusive, eq.(6).

References